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Anti Fuzzy k-bi-ideals of Γ-near-ring

N.Meena Kumari M.Sc., M.Phil., Ph.D.,
A.P.C Mahalaxmi College,Thoothukudi
V.Brishni M.Sc., M.Phil.,
V.O. Chidambaram College, Thoothukudi

Abstract

In this paper, We studied the characteristics and properties of an anti fuzzy k-bi-ideals in Γ-near-ring. We also proved the arbitrary union and non empty intersection of an anti fuzzy k-bi-ideals in Γ-near-ring is an anti fuzzy k-bi-ideals in Γ-near-ring and also complement of an anti fuzzy k-bi-ideals in Γ-near-ring is a fuzzy k-bi-ideals in Γ-near-ring.

Introduction

We discuss about the anti fuzzy k-bi-ideals in Γ-near-ring. Fuzzy k-ideals and anti fuzzy k-ideals in Γ-semi-near-ring were introduced by S.J.Alandkar and Y.S Pawar. In this chapter, we introduce the notion an anti fuzzy k-bi-ideal in a Γ-near-ring and some properties are investigated.

Preliminaries

Definition 1.1

A non-empty set N with two binary operations " + " (addition) and " · " (multiplication) is called a **near-ring**, if it satisfies the following axioms:

(i) (N, +) is a group,

(ii) (N, ·) is a semigroup,

(iii) $(x + y) \cdot z = x \cdot z + y \cdot z$ for all $x, y, z \in N$.

Precisely speaking it is a right near-ring, because it satisfies the right distributive law. We will use the word "near-ring" to mean "right near-ring". We denote xy instead of $x \cdot y$. Moreover, a near-ring N is said to be a zero-symmetric if $x \cdot 0 = 0$ for all $x \in N$, where 0 is the additive identity in N.

Definition 1.2

Let (M, +) be a group and Γ be a non-empty set. Then M is said to be a **Γ - near-ring**, if there exist a mapping $M \times \Gamma \times M \rightarrow M$ (The image of (x, α, y) is denoted by xαy) satisfying the following conditions:

(i) $(x + y)\alpha z = x\alpha z + y\alpha z$,

(ii) $(x\alpha y)\beta z = x\alpha(y\beta z)$ for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$.

Definition 1.3

Let M be a Γ-near-ring and μ be a fuzzy subset of M. Then the **complement** of μ is denoted by μ^c and is defined by $\mu^c(x) = 1-\mu(x)$ for any x ∈ M.

Definition 1.4

Let M and N be Γ-near-rings. A map f : M → N is called a Γ-near-ring **homomorphism**, if f(x + y) = f(x) + f(y) and f(xαy) = f(x)αf(y) for all x, y ∈ M and α ∈ Γ.

Definition 1.5

Let M be a Γ-near-ring. For an **endomorphism** f of M and fuzzy set μ in M, we define a new fuzzy set μ^f in M by $\mu^f(x) = \mu(f(x))$ for all x ∈ M.

Definition 1.6

Let μ be a fuzzy set of a Γ-near-ring M and f be a function defined on M, then the fuzzy set ν in f(M) is defined by $\nu(y) = \inf \{\mu(x) : x \in f^{-1}(y)$) for all y ∈ f(M) is called the **image** of μ under f. Similarly, if ν is a fuzzy set in f(M), then μ=νof in M (that is, the fuzzy set defined by μ(x) = ν(f(x))) for all x ∈ M is called the **pre-image** of ν under f.

Definition 1.7

For a family of anti fuzzy sets $\{\mu_i : i \in \wedge\}$ in a Γ-near-ring M, the **union** $\cup_{i\in\wedge} \mu_i(x)$ of $\{\mu_i : i \in \wedge\}$ is defined by $\cup_{i\in\wedge} \mu_i(x) = \sup\{\mu_i(x) : i \in \wedge\}$ for each x ∈ M.

Definition 1.8

For a family of anti fuzzy sets $\{\mu_i : i \in \wedge\}$ in a Γ-near-ring M, the **intersection** $\cap_{i\in\wedge} \mu_i$ of $\{\mu_i : i \in \wedge\}$ is defined by $\cap_{i\in\wedge} \mu_i(x)= \inf \{\mu_i(x) : i \in \wedge\}$ for each x ∈ M.

Definition 1.9

A subgroup B of (M, +) is a **ideal** if and only if MΓB ⊆ B and BΓM ⊆ B

Definition 1.10

A subgroup B of (M, +) is a **bi-ideal** if and only if BΓMΓB ⊆ B .

Definition 1.11

A bi-ideal B of M is said to be **k-bi-ideal** if y ∈ B and if x+y ∈ B or y+x ∈ B then x ∈ B.

Definition 2.1.1

A fuzzy set μ in a Γ-near-ring M is called a **fuzzy left (resp. right) ideal** of M if

(i) $\mu(x - y) \geq \min\{\mu(x), \mu(y)\}$,

(ii) $\mu(y + x - y) \geq \mu(x)$, for all x, y ∈ M.

(iii) $\mu(u\alpha(x + v) - u\alpha v) \geq \mu(x)$ (resp. $\mu(x\alpha u) \geq \mu(x)$) for all x, u, v ∈ M and α ∈ Γ.

Definition 3.1.1

A fuzzy set μ in a Γ-near-ring M is called an anti fuzzy left (resp. right) ideal of M, if

(i) $\mu(x - y) \leq \max\{\mu(x), \mu(y)\}$,

(ii) $\mu(y + x - y) \leq \mu(x)$ for all $x, y \in M$,

(iii) $\mu(u\alpha(x + v) - u\alpha v) \leq \mu(x)$(resp. $\mu(x\alpha u) \leq \mu(x)$) for all $x, u, v \in M$ and $\alpha \in \Gamma$.

Definition 2.1

An anti fuzzy bi-ideal μ in M is called an fuzzy k-bi-ideals of M if for all

$x, y \in M$, $\mu(x) \geq \min\{\max\{\mu(x+y), \mu(y+x)\}, \mu(y)\}$

Anti Fuzzy k-bi-ideals of Γ-near-ring

Definition 2.1

An anti fuzzy bi-ideal μ in M is called an anti fuzzy k-bi-ideals of M if for all $x, y \in M$, $\mu(x) \leq \max\{\min\{\mu(x+y), \mu(y+x)\}, \mu(y)\}$

Theorem 2.2

Let M be a Γ-near-ring. Then a fuzzy set μ is an anti fuzzy k-bi-ideal of M if and only if μ^c is a fuzzy k-bi-ideal of M.

Proof

Let $x, y \in M$ and μ be an anti fuzzy k-bi-ideal of M, then we have

$$\textbf{(i) } \mu^c(x - y) = 1 - \mu(x - y)$$
$$\geq 1 - \max\{\mu(x), \mu(y)\}$$
$$= \min\{1 - \mu(x), 1 - \mu(y)\}$$
$$= \min\{\mu^c(x), \mu^c(y)\},$$

For all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$, we have

$$\textbf{(ii) } \mu^c(x\alpha y\beta z) = 1 - \mu(x\alpha y\beta z)$$
$$\geq 1 - \max\{\mu(x), \mu(z)\}$$
$$= \min\{1 - \mu(x), 1 - \mu(z)\}$$
$$= \min\{\mu^c(x), \mu^c(z)\},$$

For all $x, y \in M$,

$$\textbf{(iii) } \mu^c(x)=1- \mu(x)$$
$$\geq 1 - \max\{\min\{\mu(x+y), \mu(y+x)\}, \mu(y)\}$$
$$=\min\{\max\{1-\mu(x+y), 1-\mu(y+x)\},1-\mu(y)\}$$
$$= \min\{\max\{\mu^c(x+y), \mu^c(y+x)\}, \mu^c(y)\}$$

Hence μ^c is a fuzzy k-bi-ideal of M.

Conversely, Suppose that μ^c is a fuzzy k-bi-ideal of M, then for all x, y ∈ M we have

$$\textbf{(i)}\ \mu(x-y) = 1-\mu^c(x-y) \le 1-\min\{\mu^c(x), \mu^c(y)\} = \max\{1-\mu^c(x), 1-\mu^c(y)\} = \max\{\mu(x), \mu(y)\}.$$

For all x, y, z ∈ M and α,β ∈ Γ, we have

$$\textbf{(ii)}\ \mu(x\alpha y\beta z) = 1-\mu^c(x\alpha y\beta z) \le 1-\min\{\mu^c(x), \mu^c(z)\} = \max\{1-\mu^c(x), 1-\mu^c(z)\} = \max\{\mu(x), \mu(z)\}.$$

For all x, y ∈ M

$$\textbf{(iii)}\ \mu(x)=1-\mu^c(x) \le 1-\min\{\max\{\mu^c(x+y), \mu^c(y+x)\}, \mu^c(y)\} = \max\{\min\{1-\mu^c(x+y), 1-\mu^c(y+x)\}, 1-\mu^c(y)\} = \max\{\min\{\mu(x+y), \mu(y+x)\}, \mu(y)\}$$

Hence μ be an anti fuzzy k-bi-ideal of M.

Theorem 2.3

If $\{\mu_i : i \in \wedge\}$ is a family of anti fuzzy k-bi-ideals of a Γ-near-ring M then so is $\cup_{i\in\wedge} \mu_i$.

Proof

Let $\{\mu_i : i \in \wedge\}$ be a family of anti fuzzy k-bi-ideals of M and let x, y ∈ M.

Then

$$\textbf{(i)}\cup_{i\in\wedge} \mu_i(x-y)=\sup\{\mu_i(x-y) : i \in \wedge\} \le \sup\{\max\{\mu_i(x), \mu_i(y)\}: i \in \wedge\} = \max\{\sup\{\mu_i(x) : i \in \wedge\}, \sup\{\mu_i(y) : i \in \wedge\}\} = \max\{\cup_{i\in\wedge} \mu_i(x), \cup_{i\in\wedge} \mu_i(y)\}.$$

For all x, y, z ∈ M and α, β ∈ Γ,

$$\textbf{(ii)} \cup_{i\in\wedge} \mu_i(x\alpha y\beta z) = \sup\{\mu_i(x\alpha y\beta z) : i \in \wedge\} \le \sup\{\max\{\mu_i(x), \mu_i(z)\}: i \in \wedge\} = \max\{\sup\{\mu_i(x) : i \in \wedge\}, \sup\{\mu_i(z) : i \in \wedge\}\} = \max\{\cup_{i\in\wedge} \mu_i(x), \cup_{i\in\wedge} \mu_i(z)\}.$$

For all x, y ∈ M

(iii) $\cup_{i\in\wedge} \mu_i(x)$= sup{ $\mu_i(x)$: i ∈ ∧ }

$\leq$ sup{max{min{$\mu_i(x+y)$, $\mu_i(y+x)$}, $\mu_i(y)$}: i ∈ ∧}

=max{min{sup{$\mu_i(x+y)$: i ∈ ∧},{sup{$\mu_i(y+x)$: i ∈ ∧}} ,sup{$\mu_i(y)$: i ∈ ∧}}

= max{min{$\cup_{i\in\wedge} \mu_i(x+y)$, $\cup_{i\in\wedge} \mu_i(y+z)$}, $\cup_{i\in\wedge} \mu_i(y)$}

Hence $\cup_{i\in\wedge} \mu_i$ is an anti fuzzy k-bi-ideal of a Γ-near-ring M.

Theorem 2.4

Intersection of a non empty collection of anti fuzzy k-bi-ideal of a Γ-near-ring M is an anti fuzzy k-bi-ideal of M.

Proof

Let M be a Γ-near-ring. Let {μ_i : i ∈ I} be the family of anti fuzzy k-bi-ideals of M and let x, y ∈ M.

Then we have, **(i)** $\cap_{i\in\wedge} \mu_i(x-y)$ = inf {$\mu_i(x-y)$: i ∈ ∧}

$\leq$ inf {max{$\mu_i(x)$, $\mu_i(y)$}: i ∈ ∧}

= max{inf $\mu_i(x)$: i ∈ ∧, inf $\mu_i(y)$: i ∈ ∧]

= max{ $\cap_{i\in\wedge} \mu_i(x)$, $\cap_{i\in\wedge} \mu_i(y)$}.

For all x, y, z ∈ M and α, β ∈ Γ,

we have **(ii)** $\cap_{i\in\wedge} \mu_i(x\alpha y\beta z)$= inf {$\mu_i(x\alpha y\beta z)$: i ∈ ∧}

$\leq$ inf {max{$\mu_i(x)$, $\mu_i(z)$}: i ∈ ∧ }

= max{inf $\mu_i(x)$: i ∈ ∧, inf $\mu_i(z)$: i ∈ ∧]

= max{ $\cap_{i\in\wedge} \mu_i(x)$, $\cap_{i\in\wedge} \mu_i(z)$}.

For all x, y ∈ M

(iii) $\cap_{i\in\wedge} \mu_i(x)$= inf{$\mu_i(x)$: i ∈ ∧ }

$\leq$ inf{max{min{$\mu_i(x+y)$, $\mu_i(y+x)$}, $\mu_i(y)$}: i ∈ ∧}

=max{min{inf{$\mu_i(x+y)$: i ∈ ∧},{inf{$\mu_i(y+x)$: i ∈ ∧}} ,inf{$\mu_i(y)$: i ∈ ∧}}

= max{min{$\cap_{i\in\wedge} \mu_i\ (x+y)$, $\cap_{i\in\wedge} \mu_i\ (y+z)$}, $\cap_{i\in\wedge} \mu_i\ (y)$}

Thus $\cap_{i\in\wedge} \mu_i$ is an anti fuzzy k-bi-ideal of a Γ-near-ring M.

Theorem 2.5

Γ near-ring homomorphic pre image of an anti fuzzy k-bi-ideal is an anti fuzzy k-bi-ideal.

Proof

Let $\theta : M \to N$ be a Γ near-ring homomorphism, ν be an anti fuzzy k-bi-ideal of M and μ be the pre-image of ν under θ. Then

$$\begin{aligned}\mu(x-y) &= \nu(\theta(x-y))\\ &= \nu(\theta(x))-\nu(\theta(y))\\ &\leq \max\{\nu(\theta(x),\nu(\theta(y)\}\\ &= \max\{\mu(x),\mu(y)\}.\end{aligned}$$

Further $\mu(x\alpha y\beta z) = \nu(\theta(x\alpha y\beta z))$

$$\begin{aligned}&= \nu(\theta(x)\alpha\theta(y)\beta\theta(z))\\ &\leq \max\{\nu(\theta(x)), \nu(\theta(z))\}\\ &= \max\{\mu(x), \mu(z)\} \text{ for all } x, y, z \in M \text{ and } \alpha, \beta \in \Gamma.\end{aligned}$$

And also $\mu(x)= \nu(\theta(x))$

$$\begin{aligned}&\leq \max\{\min\{\nu(\theta(x)+ \theta(y)), \nu(\theta(y)+ \theta(x))\}, \nu(\theta(y))\}\\ &= \max\{\min\{\nu(\theta(x+y)), \nu(\theta(y+x))\}, \nu(\theta(y))\}\\ &= \max\{\min\{\mu(x+y), \mu(y+x)\}, \mu(y)\}\end{aligned}$$

Hence μ is an anti fuzzy k-bi-ideal of M.

Theorem 2.6.

Let μ be an anti fuzzy k-bi-ideal of a Γ-near-ring M with an multiplicative identity e and μ^+ be a fuzzy set in M given by $\mu^+(x) = \mu(x) + 1 - \mu(e)$ for all $x \in M$. Then μ^+ is an anti fuzzy k-bi-ideal of M.

Proof

Let μ be an anti fuzzy k-bi-ideal of a Γ-near-ring M.

For all $x, y \in M$, we have

(i) $$\begin{aligned}\mu^+(x - y) &= \mu(x - y) + 1 - \mu(e)\\ &\leq \max\{\mu(x), \mu(y)\} + 1 - \mu(e)\\ &= \max\{\mu(x) + 1 - \mu(e), \mu(y) + 1 - \mu(e)\}\\ &= \max\{\mu^+(x), \mu^+(y)\}.\end{aligned}$$

For all $x, y, z \in M$ and $\alpha,\beta \in \Gamma$, we have

(ii) $$\begin{aligned}\mu^+(x\alpha y\beta z) &= \mu(x\alpha y\beta z) + 1 - \mu(e)\\ &\leq \max\{\mu(x), \mu(z)\} + 1 - \mu(e)\\ &= \max\{\mu(x) + 1 - \mu(e), \mu(z) + 1 - \mu(e)\}\end{aligned}$$

$= \max\{\mu^{+}(x), \mu^{+}(z)\}$.

For all x, y ∈ M

(iii) $\mu^{+}(x) = \mu(x) + 1 - \mu(e)$

$\leq \max\{\min\{\mu(x+y), \mu(y+x)\}, \mu(y)\} + 1 - \mu(e)$

$= \max\{\min\{\mu(x+y) + 1 - \mu(e), \mu(y+x)+ 1 - \mu(e)\}, \mu(y)+ 1 - \mu(e)\}$

$= \max\{\min\{\mu^{+}(x+y), \mu^{+}(y+x)\}, \mu^{+}(y)\}$

Hence μ^{+} is an anti fuzzy k-bi-ideal of M.

Theorem 2.7

Let M be a Γ-near-ring. Let f be an endomorphism of M. If μ is an anti fuzzy bi ideal of M, then so is μ^{f} .

Proof

Let μ is an anti fuzzy bi ideal of M.

For all x, y ∈ M and α ∈ Γ. Then

(i) $\mu^{f}(x - y) = \mu(f(x - y))$

$= \mu(f(x) - f(y))$

$\leq \max\{\mu(f(x)), \mu(f(y))\}$

$= \max\{\mu^{f}(x), \mu^{f}(y)\}$.

For all x, y,z ∈ M and α,β ∈ Γ, we have

(ii) $\mu^{f}(x\alpha y\beta z) = \mu(f(x\alpha y\beta z))$

$= \mu(f(x)\alpha f(y)\beta f(z))$

$\leq \max\{\mu(f(x)), \mu(f(z))\}$

$= \max\{\mu^{f}(x), \mu^{f}(z)\}$.

For all x, y ∈ M

(iii) $\mu^{f}(x)= \mu(f(x))$

$\leq \max\{\min\{ \mu(f(x)+ (f(y)), \mu(f(y)+ (f(x))\}, \mu(f(y)\}$

$= \max\{\min\{\mu(f(x+y)), \mu(f(y+x))\}, \mu(f(y)\}$

$= \max\{\min\{\mu^{f}(x+y), \mu^{f}(y+x)\}, \mu^{f}(y)\}$

Hence μ^{f} is an anti fuzzy k-bi-ideal of M.

Theorem 2.8

Let μ be an anti fuzzy k-bi-ideal of a Γ near-ring M.

Then $M_{\mu} = \{x \in M/\mu(x) = \mu(0)\}$ is a k-bi-ideal of M.

Proof

Let μ be an anti fuzzy k-bi-ideal of M

(i) Let $x, y \in M_\mu$.

Then $\mu(x-y) \leq \max\{\mu(x), \mu(y)\} = \mu(0)$ and so $\mu(x-y) = \mu(0) \Rightarrow x - y \in M_\mu$.

(ii) For every $x, z \in M_\mu$, $y \in M$ and $\alpha, \beta \in \Gamma$,

$\mu(x\alpha y\beta z) \leq \max\{\mu(x), \mu(z)\} = \mu(0)$.

Therefore $\mu(x\alpha y\beta z) = \mu(0) \Rightarrow x\alpha y\beta z \in M_\mu$.

(iii) For if $y \in M_\mu$ and if $x+y \in M_\mu$ or $y+x \in M_{\mu,}$

Then $\mu(y) = \mu(0)$ and either $\mu(x+y)= \mu(0)$ or $\mu(y+x) = \mu(0)$

Therefore $\mu(x) \leq \max\{\min\{\mu(x+y), \mu(y+x)\}, \mu(y)\} = \mu(0) \Rightarrow x \in M_\mu$

Hence M_μ is a k-bi-ideal of M.

Theorem 2.9

Let A be a non-empty subset of a Γ near-ring M and μ_A be a fuzzy set in M defined by

$$\mu_A(x) = \begin{cases} s \ if \ x \in A \\ t \ if \ x \notin A \end{cases}$$

for all $x \in M$ and $s, t \in [0, 1]$ with $s<t$. Then μ_A is an anti fuzzy k-bi-ideal of M if and only if A is a k-bi-ideal of M. Moreover $M\mu_A = A$.

Proof

Let μ_A be an anti fuzzy k-bi-ideal of M and let $x, y \in A$.

Then $\mu_A(x-y) \leq \max\{\mu_A(x), \mu_A(y)\}$

$= s$

and so $\mu_A(x - y) = s$ implies $x - y \in A$.

For every $x, z \in A$, $y \in M$ and $\alpha, \beta \in \Gamma$,

$\mu_A(x\alpha y\beta z) \leq \max\{\mu_A(x), \mu_A(z)\} = s$

and so $\mu_A(x\alpha y\beta z) = s$ implies $x\alpha y\beta z \in A$.

For if $y \in A$ and if $x+y \in A$ or $y+x \in A$

Then $\mu_A(y) = s$ and either $\mu_A(x+y) = s$ or $\mu_A(y+x) = s$

$\mu_A(x) \leq \max\{\min\{\mu_A(x+y), \mu_A(y+x)\}, \mu_A(y)\} = s$

and so $\mu_A(x) = s$ implies $x \in A$.

Hence A is a k-bi-ideal of M.

Conversely, assume that A is a k-bi-ideal of M.

Let $x, y \in M$.

If at least one of x or y is not in A, then $\mu_A(x - y) = t$

$$\leq \max\{\mu_A(x), \mu_A(y)\}.$$

If $x, y \in A$, then $x - y \in A$ and so $\mu_A(x - y) = s$

$$= \max\{\mu_A(x), \mu_A(y)\}.$$

Let $x, y, z \in M$ and $\alpha, \beta \in \Gamma$.

(a) If $x, z \in A$, then $x\alpha y\beta z \in A$ and so $\mu_A(x\alpha y\beta z) = s$

$$\leq \max\{\mu_A(x), \mu_A(z)\}.$$

(b) If $x \in A$, $z \notin A$, then $\mu_A(x\alpha y\beta z) = t \leq \max\{\mu_A(x), \mu_A(z)\}$.

(c) If $x \notin A$, $z \in A$, then $\mu_A(x\alpha y\beta z) = t \leq \max\{\mu_A(x), \mu_A(z)\}$.

(d) If $x \notin A$, $z \notin A$, then $\mu_A(x\alpha y\beta z) = t = \max\{\mu_A(x), \mu_A(z)\}$.

Thus μ_A is an anti fuzzy bi-ideal of M.

Let $x, y \in M$.

If at least one of x or y is not in A, then $\mu_A(x + y) = t$ and $\mu_A(y + x) = t$

And so $\max\{\min\{\mu_A(x+y), \mu_A(y+x)\}, \mu_A(y)\}$

$$\begin{aligned} \text{Now } M\mu_A &= \{x \in M / \mu_A(x) = \mu_A(0)\} \\ &= \{x \in M / \mu_A(x) = s\} \\ &= \{x \in M / x \in A\} \\ &= A. \end{aligned}$$

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